Q.P. Code – 50825

Third Year B.Sc. Degree Examination

OCTOBER/NOVEMBER 2014

(Directorate of Distance Education)

(DSC 232) Paper V – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

*Answer any **SIX** choosing atleast TWO questions from each Part.*

PART – A

1. (a) (i) Find the real and imaginary parts of e^{z^2}. **2**

(ii) Find whether $f(z) = \bar{z}$ is differentiable or not. **2**

(b) Show that $\arg\left[\frac{z-1}{z+1}\right] = \frac{\pi}{3}$ represents a circle and find its centre and radius. **5**

(c) Find the equation of the circle passing through the points, $1+i$, $1-i$ and 2 and find its centre and radius. **6**

2. (a) (i) Verify whether $f(z) = \sin z$ is analytic or not. **2**

(ii) If $f(z)$ is analytic function such that $f(z)$ is always real. Show that f is constant. **2**

(b) Show that $u = \cos x \cosh y$ is harmonic and find the analytic function whose real part is $u(x, y)$. **5**

(c) Show that two functions $u(x, y)$ and $v(x, y)$ are harmonic conjugate of each other if and only if they are constant functions. **6**

3. (a) (i) Evaluate $\int_C (3x+y)dx+(2y-x)dy$ along the line from (0, 1) to (0, 5) and then from (0, 5) to (2, 5). **2**

(ii) Find the fixed point of the bilinear transformation $w=\frac{z-1}{z+1}$. **2**

(b) If $f(z)$ be analytic within and on a closed contour c of a simple connected region and $z=z_0$ is an interior point of c then prove that $\frac{1}{2\pi i}\int_c \frac{f(z)}{z-z_0}dz=f(z_0)$. **5**

(c) Discuss the Transformation $w=\sin z$. **6**

4. (a) (i) Prove that $E=1-\Delta$. **2**

(ii) Evaluate Δx^3, taking $h=1$. **2**

(b) From the following table find the number of students who obtain less than 45 marks : **5**

No. of students :	31	42	51	35	31
Marks :	30-40	40-50	50-60	60-70	70-80

(c) Find $f'(x)$ and $f''(x)$ of the function $f(x)$ at $x=1.5$. Given **6**

x	1.5	2	2.5	3	3.5
$f(x)$	3.375	7	13.625	24	38.875

PART – B

5. (a) (i) Find $L[\sinh mt]$. **2**

(ii) Find $L[\sin 3t \cdot \cos 4t]$. **2**

(b) Find the Laplace transform of the periodic function

$$f(t)=\begin{cases} 1 & \text{for } 0<t<\frac{a}{2} \\ -1 & \text{for } \frac{a}{2}<t<a \end{cases}$$

and $f(t+a)=f(a)$. **5**

(c) Express $f(t)$ in terms of unit-step function and find $L[f(t)]$ where

$$f(t)=\begin{cases} 2t & 0<t<\pi \\ 1 & t>\pi. \end{cases}$$ **6**

6. (a) (i) Find $L\left[t\,e^{at}\sin t\right]$. **2**

(ii) Find $L^{-1}\left\{\log\left[\frac{s^2+1}{(s-1)^2}\right]\right\}$. **2**

(b) Evaluate $L^{-1}\left[\frac{1}{(s+2)(s+4)}\right]$ using Convolution theorem. **5**

(c) Solve $9y''-6y'+y=0$ given $y(0)=3$ and $y'(0)=1$. **6**

7. (a) (i) Evaluate $\left[\frac{\Delta^2}{E}\right]e^x\cdot\frac{Ee^x}{\Delta^2 e^x}$ taking h as the interval of differencing. **2**

(ii) Find $\Delta[\log(ax+b)]$. **2**

(b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$, using Trapezoidal Rule with $h=0.2$. **5**

(c) Using Simpson's $\frac{3}{8}$th rule to obtain the approximate value of $\int_0^{0.3}\left(2x-x^2\right)^{1/2}dx$ by taking $n=6$. **6**

8. (a) (i) Using Weedle's Rule evaluate $\int_0^1 \frac{1}{1+x}dx$. Given that **2**

x	0	1/6	2/6	3/6	4/6	5/6	6/6
y	1	6/7	6/8	6/9	6/10	6/11	6/12

(ii) Show that the real root of x^3-9x+1 lies between 2 and 4. **2**

(b) Solve $x^3+2x^2+10x-20=0$ by Newton Raphson's method, correct to four decimal places given that root lies near $x=1.5$. **5**

(c) Solve : $\frac{dy}{dx}=1+\frac{y}{x}$ given that $y=2.0$ when $x=2.0$ and $h=0.2$ using Runge-Kutta 4th order method. **6**
