

Q.P. Code – 50622

First Year B.Sc. Degree Examination

OCTOBER/NOVEMBER 2014

(Directorate of Distance Education)

(DSA 230) Paper I – MATHEMATICS

Time : 3 Hours]

[Max. Marks : 90

Instructions to Candidates :

*Answer any **SIX** full questions of the following choosing atleast **ONE** from each Part.*

PART – A

1. (a) (i) Find the last digit in 7^{1011} . **2**
- (ii) Solve the congruence $3x \equiv 4 \pmod{5}$. **2**
- (b) Solve the simultaneous congruences
- $$3x \equiv 2 \pmod{5}$$
- $$2x \equiv 1 \pmod{3}$$
- 5**
- (c) State and prove Chinese Remainder theorem. **6**
2. (a) (i) A function $f : R \rightarrow R$ is defined by $f(x) = 3x + 1$. Prove that f is a bijective function. **2**
- (ii) Let $f : R \rightarrow R$ and $g : R \rightarrow R$ be defined by $f(x) = 4x$ and $g(x) = \cos x$. Find $f \circ g$ and $g \circ f$. **2**
- (b) Given $f : A \rightarrow B$ is defined by $f(x) = \frac{x+5}{x+4}$ when $A = R - \{-4\}$, $B = R - \{1\}$. Show that f^{-1} exists and find a formula for f^{-1} . **5**
- (c) Prove that the set of integers z is countable. **6**

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PART – B

3. (a) (i) Show that the function $f(x) = |x|$ at $x = 0$ has no derivative. **2**
- (ii) Find n th derivative of $e^x \sin^2 x$. **2**
- (b) Examine the differentiability of
- $$f(x) = \begin{cases} 1 + \sin x & \text{if } x < \frac{\pi}{2} \\ 2 + \left(x - \frac{\pi}{2}\right)^2 & \text{if } x \geq \frac{\pi}{2} \end{cases} \quad \text{at } x = \frac{\pi}{2}. \quad \mathbf{5}$$
- (c) If $y = \frac{\sinh^{-1} x}{\sqrt{1+x^2}}$ prove that $(1+x^2)y_{n+2} + (2n+3)xy_{n+1} + (n+1)^2y_n = 0$. **6**
4. (a) (i) Find $\frac{dS}{d\theta}$ for the curve $r = a \cos \theta$. **2**
- (ii) Find the radius of curvature for the curve $y = e^x$ at $x = 0$. **2**
- (b) Find the Pedal equation for the curve $r^n = a^n \operatorname{sech} n\theta$. **5**
- (c) Find the angle of intersection of the pair of curves $r = \frac{a\theta}{1+\theta}$, $r = \frac{a}{1+\theta^2}$. **6**

PART – C

5. (a) (i) Does the line through the points $(-1, 1, 2)$ and $(3, 2, 4)$ cuts the x -axis. **2**
- (ii) If the planes $x - y = 2$ and $3x - ay = 7$ are parallel, find a . **2**
- (b) Find the reflection of the point $P:(4, -2, 3)$ in the line $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$. **5**
- (c) Find the coordinates of the point in which the line $\frac{x+3}{4} = \frac{y+4}{3} = \frac{z-8}{-5}$ meets the sphere $x^2 + y^2 + 2x - 10y - 23 = 0$. **6**

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6. (a) (i) Define Asymptotes. **2**
(ii) Find the asymptotes of the curve $r\theta = a$. **2**
(b) Find the position and nature of the double point of the curve $x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$. **5**
(c) Find the area of the loop of the curve $ay^2 = x^2(a - x)$. **6**

PART – D

7. (a) (i) Express the matrix $A = \begin{bmatrix} 1 & 4 \\ 7 & 2 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices. **2**

- (ii) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 2 & 4 & 6 \end{bmatrix}$. **2**

- (b) Find the inverse of the following matrix $A = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$. **5**

- (c) Find the value of λ for which the following system has a non-trivial solution :

$$2x - y + 2z = 0$$

$$3x + y - z = 0$$

$$\lambda x - 2y + z = 0$$

6

8. (a) (i) Evaluate $\int \sqrt{1 + \sin x} dx$. **2**

- (ii) Evaluate $\int \frac{x e^x}{(1+x)^2} dx$. **2**

- (b) Evaluate $\int_0^{\pi/6} \cos^4 3x \sin^2 6x dx$ using reduction formula. **5**

- (c) Evaluate $\int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}}$. **6**