Q.P. Code – 50825

Third Year B.Sc. Degree Examination

SEPTEMBER/OCTOBER 2013

(Directorate of Distance Education)

(DSC 232) Paper V – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

*Answer any **SIX**, choosing atleast **TWO** questions from each Part.*

PART – A

1. (a) (i) Find the real and imaginary parts of $\exp\left[7+i\frac{\pi}{2}\right]$. **2**

(ii) Evaluate $\lim_{z \to i}\left[\frac{z^2+1}{z^6+1}\right]$. **2**

(b) Show that $\arg\left[\frac{z-1}{z+1}\right]=\frac{\pi}{3}$ represents a circle. **5**

(c) Find the equation of the circle passing through the points, 1, i and $1+i$. Find its Centre and Radius. **6**

2. (a) (i) Show that $f(z)=\bar{z}$ is not differentiable anywhere. **2**

(ii) If $f(z)$ is analytic in an open set S and $f'(z)=0$ for all $z \in S$, show that $f(z)$ is constant. **2**

(b) Show that $f(z)=\log z$ is analytic and hence $f'(z)=\frac{1}{z}$. **5**

(c) If $f(z)=u+iv$ is analytic and ϕ is any differentiable function of x and y show that $\left(\frac{\partial \phi}{\partial x}\right)^2+\left(\frac{\partial \phi}{\partial y}\right)^2=\left[\left(\frac{\partial \phi}{\partial u}\right)^2+\left(\frac{\partial \phi}{\partial v}\right)^2\right]|f(z)|^2$. **6**

3. (a) (i) Evaluate $\int_C \left[x^2 - iy^2\right]dz$ along $y = 2x^2$ from (1, 2) to (2, 8). **2**

(ii) Find the fixed points of the bilinear transformation $w = \frac{i - z}{z + i}$. **2**

(b) State and prove Cauchy's integral theorem. **5**

(c) Discuss the Transformation $w = \frac{1}{2}\left[z + \frac{1}{z}\right]$. **6**

4. (a) (i) Evaluate $\Delta\left[\frac{2^x}{x!}\right]$ with $h = 1$. **2**

(ii) Evaluate $(\nabla + \Delta)^2\left(x^2 + x + 1\right)$ interval of difference is 1. **2**

(b) The population of a town is as follows : **5**

Year :	1921	1931	1941	1951	1961	1971
Population in Lakhs :	20	24	29	36	46	51

Estimate the increase in population during the period 1955 to 1961.

(c) A function is specified by the following table : **6**

x :	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y :	1.00	1.0247	1.0488	1.0723	1.0954	1.1180	1.1401

Find y' and y'' at $x = 1$.

PART – B

5. (a) (i) Find Laplace transform of t^n, where n is a positive integer. **2**

(ii) Find Laplace transform of $\sin^2 t$. **2**

(b) Find the Laplace transform of the function **5**

$$f(t) = \begin{cases} E & \text{for } 0 \le t \le T/2 \\ -E & \text{for } \frac{\pi}{2} \le t \le T \end{cases}$$

(c) Express $f(t)$ in terms of unit-step function and find $L\{f(t)\}$ where

$$f(t) = \begin{cases} t^2 & \text{where } 0 < t < 2 \\ t - 1 & \text{where } 2 < t < 3 \\ 7 & \text{where } t > 3 \end{cases}$$ **6**

6. (a) (i) Find $L^{-1}\left\{\frac{S+2}{S^2-4S+13}\right\}$. **2**

(ii) Find $L^{-1}\left\{\frac{2S}{(S^2-4)^2}\right\}$. **2**

(b) Find the inverse transform of the function using Convolution theorem $\frac{S}{(S^2+a^2)^2}$. **5**

(c) Solve by using Laplace transform $y''+9y=25e^{4t}$ given $y(0)=3$ and $y'(0)=7$. **6**

7. (a) (i) Find value of $\Delta e^{2x}\cdot\log 3x$. **2**

(ii) Evaluate $\left[\frac{\Delta^2}{E}\right]e^x\cdot\frac{Ee^x}{\Delta^2 e^x}$. **2**

(b) Evaluate $\int_4^{5.2}\log x$ by using Trapezoidal Rule. **5**

(c) Using Simpson's $\frac{1}{3}$rd rule evaluate $\int_0^3\frac{dx}{1+x}$ by dividing the interval (0, 3) into six equal parts. **6**

8. (a) (i) Evaluate $\int_0^6\frac{1}{1+x^2}dx$ using Weedle's Rule. **2**

(ii) Find a real root of the equation $x^2-9x+1=0$. Given that the root lies between 2 and 4. **2**

(b) Using Newton Raphson method, find the real root of the equation $\tan x = x$, near $x=4.5$, correct to four decimal places. **5**

(c) Find the approximate solution at $x=1.2$ of the equation $\frac{dy}{dx}=xy$ given $y(1)=2$ by Runge-Kutta method. **6**
