Q.P. Code – 50823

Third Year B.Sc. Degree Examination

SEPTEMBER/OCTOBER 2013

(Directorate of Distance Education)

(DSC 230) Paper III – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

*Answer any **SIX** full questions of the following choosing atleast **ONE** from each Part.*

PART – A

1. (a) (i) Prove that the intersection of any two normal subgroups of a group G is also a normal subgroup. **2**

(ii) Show that every quotient group of an abelian group is abelian. **2**

(b) Prove that a subgroup H of a group G is a normal subgroup of G if and only if $gHg^{-1} \in H \;\forall\; g \in G$. **5**

(c) State and prove the fundamental theorem on homomorphism of groups. **6**

2. (a) (i) Define ring without zero divisors and give an example. **2**

(ii) Show that the set $S = \left\{ \begin{bmatrix} a & 0 \\ b & c \end{bmatrix} : a, b, c \in z \right\}$ is a subring of the ring $M_2(z)$ of all 2 × 2 matrices over the set of integers. **2**

(b) If U and V are ideals of a ring R, then show that $U + V$ is also an ideal of R. **5**

(c) If p is an integer then prove that pz is a maximal ideal of $(z, +, X)$ if and only if p is prime. **6**

PART – B

3. (a) (i) Is $f(x) = x^2 + x + 2$ is reducible over z_5. **2**

(ii) Find all the units and zero divisors of a commutative ring z_8. **2**

(b) Find the GCD of $f(x) = 4x^3 - 12x^2 - 15x - 4$ and $g(x) = 12x^2 - 24x - 15$ over $Q[x]$. **5**

(c) If $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ is a polynomial in x of degree n with integral coefficients then prove that any rational root of the equation $f(x) = 0$ must have the form $\frac{r}{s}$ where r/a_0 and s/a_n. **6**

4. (a) (i) Prove that the centre z of a group G is a normal subgroup of G. **2**

(ii) Show that z is not an ideal of the ring $(Q, +, \cdot)$ **2**

(b) Let G be group of all real numbers w.r.t. '+' and G' be group of non-zero complex numbers w.r.t. 'X'. Show that $f : G \to G'$ defined by $f(x) = \cos x + i \sin x$ is a homomorphism. Find its Kernel. Is f an isomorphism? **5**

(c) If $f : R \to R'$ be a homomorphism of rings with Kernel K then prove that

(i) K is a subring of R

(ii) Kernel K of 'f' is an ideal in R. **6**

PART – C

5. (a) (i) In a vector space $V(F)$, prove that $C \cdot \alpha = \vec{0} \Rightarrow C = 0$ or $\alpha = \vec{0}$. **2**

(ii) Show that the subset $w = \{(x, y, z)/x + y + z = 0\}$ of the vector space $V_3(R)$ is a subspace of $V_3(R)$. **2**

(b) In $V_3[z_3]$ how many vectors are spanned by the vectors (1, 2, 1) and (2, 1, 2). **5**

(c) Prove that the set $S = \{\alpha_1, \alpha_2, \cdots, \alpha_n\}$ of non-zero vectors of vector space $V[F]$ with $\alpha_1 \neq 0$ is linearly dependent if and only if one of the vectors say $\alpha_K (K \geq 2)$ is a linear combination of its preceding ones. **6**

6. (a) (i) Determine whether the vectors $(1, 0, 2)$, $(-1, 0, 1)$, $(0, 1, 2)$ of $V_3(R)$ linearly dependent or linearly independent. **2**

(ii) Prove that the super set of linearly dependent set is linearly dependent. **2**

(b) Show that the set $s = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ form a basis of the vector space V of all 2×2 matrices over R. **5**

(c) Find the basis and dimension of the subspace spanned by $S = \{(1, 2, 3), (3, 1, 0), (-2, 1, 3)\}$ in $V_3(R)$. **6**

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PART – D

7. (a) (i) If $T:U \rightarrow V$ is a linear transformation then prove that $T(-\alpha) = -T(\alpha)\ \forall\ \alpha \in V$. **2**

(ii) Find the linear transformation $T: R^2 \rightarrow R^2$ such that $T(1, 1) = (0, 1)$ and $T(-1, 1) = (3, 2)$. **2**

(b) Find the range space, Kernel, rank and nullity of the linear transformation $T: V_2(R) \rightarrow V_2(R)$ defined by $T(x, y) = (x + y, x)$. Also verify rank-nullity theorem. **5**

(c) Find the orthonormal basis for the vector space spanned by the vectors $\{(1, 0, 0), (1, 1, 1), (1, 2, 3)\}$. **6**

8. (a) (i) If $u = x^2y + y^2z + z^2x$ then show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x + y + z)^2.$$ **2**

(ii) If $z = e^x(x\cos y - y\sin y)$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = 0$. **2**

(b) If $u = \sin^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, then show that $x \cdot \frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$. **5**

(c) Investigate the maximum and minimum of the function

$f(x, y) = x^3 + y^3 - 3xy$. **6**