Q.P. Code – 50622

First Year B.Sc. Degree Examination

SEPTEMBER/OCTOBER 2013

(Directorate of Distance Education)

(DSA 230) Paper I – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

*Answer any **SIX** full questions of the following choosing atleast **ONE** from each Part.*

PART – A

1. (a) (i) Find the G.C.D. of 275 and 726. **2**

(ii) If a/b and b/c then a/c. **2**

(b) Find the remainder when 2^{23} is divided by 47. **5**

(c) State and prove Chinese Remainder theorem. **6**

2. (a) (i) Define symmetric and Transitive relations. **2**

(ii) If f and g are real functions defined by $f(x)=2x^2+3x+5$ and $g(x)=4x-5$ find $f\circ g$ and $g\circ f$. **2**

(b) If $f:X\to Y$ is a function from x into y then for subsets $A, B\in X$ and $C, D\in Y$. Prove that $f^{-1}(C\cap D)=f^{-1}(C)\cap f^{-1}(D)$. **5**

(c) Find the partition of the set z of all integers defined by the equivalence relation $a\,R\,b$ iff $(a-b)$ is a multiple of 5. **6**

PART – B

3. (a) (i) Evaluate $\lim_{x\to 3} f(x)$ where $f(x)=\dfrac{x^2-9}{x-3}$ if it exists. **2**

(ii) If $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ then find $\dfrac{dy}{dx}$. **2**

(b) If $f(x)=\begin{cases} x^2-1 & \text{when } x<1 \\ 0 & \text{when } x=1 \\ 1-\frac{1}{x} & \text{when } x>1 \end{cases}$

Show that the function is continuous at $x=1$. **5**

(c) If $y=x\sin(\log x)+x\log x$ prove that $x^2y_2-xy_1+2y-x\log x=0$. **6**

4. (a) (i) Find $\frac{dS}{dx}$ for the curve $x^{2/3}+y^{2/3}=a^{2/3}$. **2**

(ii) Show that the radius of curvature of the curve $x^4+y^4=2$ at (1, 1) is $\sqrt{2}/3$. **2**

(b) Show that the Pedal equation of the parabola $y^2=4a(x+a)$ is $p^2=ar$. **5**

(c) Find the evolute of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. **6**

PART – C

5. (a) (i) Find the equation of the plane passing through the point $(0, 1, -2)$ and parallel to the plane $2x-3y+4z=0$. **2**

(ii) Show that the planes $5x+7y+2z+1=0$ and $10x+14y+4z+5=0$ are parallel. **2**

(b) Find the equation of the plane passing through the points $(2, -3, 4)$, $(1, 2, 3)$ and $(-5, 3, -1)$. **5**

(c) Determine the mutual positions of the lines l_1 and l_2 where $l_1:\frac{x-1}{-1}=\frac{y-2}{1}=\frac{z}{2}$, $l_2:\frac{x-3}{-2}=\frac{y-4}{2}=\frac{z-6}{4}$ **6**

6. (a) (i) Find the equation of the sphere whose centre is $(3, -4, 5)$ and radius is 7. **2**

(ii) Find the asymptotes parallel to the co-ordinate axes for the curve $xy^2=4a^2(2a-x)$. **2**

(b) Find all the asymptotes of the curve $x^3+y^3-3axy=0$. **5**

(c) Find the area bounded by the cardioid $r=a(1+\cos\theta)$. **6**

PART – D

7. (a) (i) Express the matrix $A=\begin{bmatrix}6 & 7\\ 9 & 4\end{bmatrix}$ as the sum of symmetric and skew symmetric matrix. **2**

(ii) Find the eigenvalues of the matrix $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$. **2**

(b) Find the Rank of the matrix $\begin{bmatrix}2 & 1 & 5 & -4\\ 3 & -2 & 2 & -2\\ 5 & -8 & -4 & 2\end{bmatrix}$. **5**

(c) Verify whether the following system of equations is consistent. If consistent solve

$$\begin{aligned} x+y+z+3&=0\\ 3x+y-2z+2&=0\\ 2x+5y+7z-7&=0\end{aligned}$$ **6**

8. (a) (i) Evaluate $\int\frac{dx}{\sec x+\tan x}$. **2**

(ii) $\int_0^{\pi}\sin^3 x\cos^2 x\,dx$ **2**

(b) Evaluate $\int\frac{\sin x}{\sin x+\cos x}dx$. **5**

(c) Evaluate $\int_0^{\pi}\frac{x}{1+\sin x}dx$. **6**