DSC – 230

Third Year B.Sc. Degree Examination, September/October 2012
(Directorate of Distance Education)
MATHEMATICS (Paper – III)

Time : 3 Hours Max. Marks : 90

***Note** : Answer **any SIX full** questions of the following choosing atleast one from **each** Part.*

PART – A

1. a) i) P.T. intersection of two normal subgroups of a group is also a normal subgroup. **2**
   ii) Define a Quotient group and binary composition of G/H. **2**
   b) A subgroup H of a group G is a normal subgroup of G iff every right coset of H in G is a left coset of H in G. **5**
   c) State fundamental theorem on homomorphism of groups. **6**

2. a) i) P.T. a ring is without zero divisors satisfies cancellation laws. **2**
   ii) Let R be a ring and ‘a’ be a fixed element of R. Show that $Ra = \{x \in R / ax = 0\}$ is a subring of R. **2**
   b) The ring $(Z_n, +_n, X_n)$ is an integral domain and hence a field, iff ‘n’ is a prime number. **5**
   c) An ideal I of a commutative ring ‘R’ with unity ‘I’ is a maximal ideal iff R/I is a field. **6**

3. a) i) Show that the polynomial $f(x) = x^3 + 3x + 2$ is irreducible over the field z_5 of integers modulo 5. **2**
   ii) Find the Zero’s of the polynomial $f(x) = x^3 + x + 2$ in z_3. **2**
   b) Find the G.C.D. of the polynomials $f(x) = x^4 + 1$, $g(x) = x^2 + x + 2$ in z_3. **5**
   c) If $f(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ is a polynomial in x of degree n with integral coefficients then prove that any rational root of the equation $f(x) = 0$ must have the form $\frac{r}{s}$ where r / a_0 and s / a_n. **6**

4. a) i) Prove that every subgroup of an abetian group is a normal subgroup. **2**
   ii) Prove that Kernel of homomorphism of a ring is a subring of R. **2**
   b) If $f : G \to G$ is a homomorphism of the group ‘G’ into itself and H is a cyclic subgroup of ‘G’, then prove that f(H) is a cyclic subgroup of G. **5**
   c) State and prove fundamental theorem of homomorphism of rings. **6**

PART – B

5. a) i) P.T. a non-empty subset W of a Vector Space V(F) is a subspace of V iff $\forall\alpha, \beta \in V,\ a, b \in F,\ a\alpha + b\beta \in W$. **2**

ii) P.T. $w = \{(x, 0, 0)/x \in R\}$ is a subspace of $V_3(R)$. **2**

b) Let $\{\alpha_1, \alpha_2 \ldots \alpha_n\}$ is linearly independent set, prove that β is a linear combination of $\alpha_1, \alpha_2 \ldots \alpha_n$ iff $(\alpha_1, \alpha_2 \ldots \alpha_n, \beta)$ is linearly dependent. **5**

c) In $V_3(R)$, Let $\alpha = (1, 2, 1)$ and $\beta = (3, 1, 5)$, $\gamma = (3, -4, 7)$ show that the subspace spanned by $S = \{\alpha, \beta\}$ and $T = \{\alpha, \beta, \gamma\}$ are same. **6**

6. a) i) Prove that the vectors (1, 0, 2), (–1, 0, 1), (0, 4, 2) are linearly independent in $V_3(R)$. **2**

ii) Show that the set containing a zero vector is linearly dependent. **2**

b) Show that $S = \left\{\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}, \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\right\}$ form a basis of a vector space of all 2×2 symmetric matrices. **5**

c) Find the basis and dimension of the subspace spanned by S = {(1, 1, 1), (2, 1, 2), (1, 0, 1), (5, 3, 5)} in $V_3(R)$. **6**

7. a) i) If $T : V \to W$ is a linear transformation, then prove that $T(-\alpha) = -T(\alpha)\ \forall \alpha \in V$. **2**

ii) Find a linear transformation $T : V_2(R) \to V_2(R)$ defined by T(1, 0) = (2, 3) and T(0, 1) = (3, –1). **2**

b) Find the matrix of linear transformation $T : V_2(R) \to V_2(R)$ defined by T(2, 3) = (1, –3), T(3, 2) = (–1, 4). **5**

c) Show that the linear transformation $T : V_2(R) \to V_2(R)$ defined by T(x, y) = (x – y, x – 2y) is non-singular and find its inverse. **6**

8. a) i) If $u = x^y$ find u_x and u_y. **2**

ii) If $u = \frac{x^2 + y^2}{xy}$, show that $xu_x + yu_y = 2u$. **2**

b) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, $x \neq y$, prove that $xu_x + yu_y = \sin 2u$. **5**

c) State and prove Euler's theorem for homogeneous functions. **6**