DSC 231

Third Year B.Sc., Degree Examination

Directorate of Distance Education

July / August 2011

MATHEMATICS - IV

Time: 3 hrs.] [Max.Marks: 90

Note :- Answer any six of the following.

PART – A

1. (a) (i) Evaluate $\int_C y dx - x\, dy - z^2\, dz$ where C is the curve $x = \sin t$, $y = \cos t$, $z = t^2, 0 \le t \le 1$ 2 Marks

ii) Evaluate $\int_0^1 \int_0^2 (x + y) dx\, dy$ 2 Marks

(b) Show that $\int_C \frac{x^2 dy - y^2 dx}{x^{5/3} + y^{5/3}} = \frac{3\pi}{16} a^{4/3}$ where C is the quarter of the asteroid $x = a\cos^3 t$, $y = a\sin^3 t$ from the point (a, 0) to the point (0, a) 5 Marks

(c) Evaluate $\iint_D (x^2 + y^2)^2\, dx\, dy$ D is the region $r = \cos 2\theta$ $-\pi/4 < \theta < \pi/4$ 6 Marks

2. (a) (i) Evaluate $\iint w \sin^{-1} z\, dz\, dw$ where $o < z < 1,\ 2 < w < 3$ 2 Marks

(ii) Evaluate $\int_0^2 \int_1^3 \int_1^2 xy^2 z\, dx\, dy\, dz$ 2 Marks

(b) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} dx\, dy\, dz$ 5 Marks

(c) Find the volume common to the cylinder $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ 6 Marks

3. (a) (i) Show that $\beta(m, n) = \beta(n, m)$ 2 Marks

(ii) Show that $\int_0^{\pi/2} \sqrt{\tan\theta}\ d\theta = \frac{r(3/4)\, r(1/4)}{2}$ 2 Marks

(b) Prove that $\beta\left(\frac{p+1}{2}, \frac{q+1}{2}\right) = 2\int_0^{\pi/2} \sin^p\theta \cos^q\theta\, d\theta$ 5 Marks

(c) Prove that $\int_0^a y^4 \sqrt{(a^2 - y^2)}\ dy = \frac{\pi a^6}{32}$ 6 Marks

4. (a) (i) If f: [a, b] → R is bounded function P ∈ P [a, b] then prove that L(p, f) ≤ U (p, f) 2 Marks

(ii) Prove that every continuous function is R – integrable 2 Marks

(b) State and prove Darbrux theorem. 5 Marks

(c) Show that $f(x) = x^2$ is R – integrable in [0,1] using lower and upper Riemann integrals and $\int_0^1 x^2 dx = \frac{1}{3}$ 6 Marks

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PART – B

5. (a) (i) Find the complementary function of $y'' + (1 - \cot x)\, y' - y \cot x = \sin^2 x$ 2 Marks

(ii) Find the Wronskian W for the equation $y'' + y = \operatorname{cosec} x$ 2 Marks

(b) Solve $y'' + (\tan x)\, y' + (4 \cos^2 x)\, y = 0$ by changing the independent variable 5 Marks

(c) Solve $x^2 y'' - 2x(1+x)\, y' + 2(1+x)\, y = x^3,\ x > 0$ by changing the dependent variable given that $y = \frac{1}{2}$ and $\frac{dy}{dx} = 1$ when $x = 1$. 6 Marks

6. (a) (i) Verify the condition of exactness of the equation $(1-x^2)\, y'' - 3xy' - y = 0$ 2 Marks

(ii) Write the complementary function for the cases $1 - P + Q = 0$ and $P + QX = 0$ 2 Marks

(b) Solve $x^2y'' + xy' - y = 2x^2, x > 0$, given that $\frac{1}{x}$ is a part of complementary function. 5 Marks

(c) Solve $(D^2 + D)y = \operatorname{cosec} x$ by the method of variation of parameters. 6 Marks

7. (a) (i) Verify the condition of integrability of the function $yz\,dx - 2xz\,dy + (xy - zy^3)\, dz = 0$ 2 Marks

(ii) Form the partial differential equation by eliminating the arbitrary constants a and b in $(x - a)^2 + (y - b)^2 + z^2 = r^2$ 2 Marks

(b) Solve $\frac{dx}{x(y^2-z^2)} = \frac{dy}{y(z^2-x^2)} = \frac{dz}{z(x^2-y^2)}$ 5 Marks

(c) Solve $P(mz - ny) + (nx - lz)\, q = (ly - mx)$ 6 Marks

8. (a) (i) Find the Fourier constants 'a_n' and 'b_n' with period 2L. 2 Marks

(ii) If $f(x) = x - x^2,\ -1 < x < 1$ find a_0 2 Marks

(b) Find the half range cosine series for $f(x) = \pi - x$ in the interval $(0, \pi)$ 5 Marks

(c) Find the Fourier series of the function $f(x) = x^2$ in the interval $-\pi < x < \pi$ and hence deduce that $\pi^2/6 = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots$ 6 Marks

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