# Directorate of Correspondence Course

## Third Year B.Sc. Degree Examinations

## August /September 2010

(New Scheme)

MATHEMATICS

Paper - V

Time: 3 hrs.] [Max.Marks : 90

Note: *Answer any SIX questions.*

PART - A

A. *Answer the following.*

1. a) i) Derive the general equation of the circle in the form $Z\bar{z} + b\bar{z} + \bar{b}\,z + c = 0$ **2 Marks**

ii) Find the derivative of $f(z) = \frac{z-1}{z+1}$ at $2 - i$. **2 Marks**

b) Evaluate $\lim_{z \to 0} \frac{x^2 y}{x^4 + y^2}$ **2 Marks**

c) Prove that the necessary condition for the equation $f(z) = u(x,y) + iv(x,y)$ to be analytic in a domain is that $U \& V$ satisfy the equations $U_x = V_y$ & $U_y = -V_x$. **6 Marks**

2. a) i) Find an analytic function $f(z)$ whose real part is $e^x \sin y$. **2 Marks**

ii) If a function $f(z)$ is defferentiable then prove that it is continuous. **2 Marks**

b) Prove that $2x - x^3 + 3xy^2$ is harmonic and find its harmonic conjugate. **5 Marks**

c) Evaluate $\int_c (x^2 - iy^2)\, dz$ along the parabola $y = 2x^2$ from $(1,2)$ to $(2,8)$ **6 Marks**

3. a) i) Evaluate $\int_c \frac{z-3}{z^2+2z+5}\, dz$ where $C$ is the circle $|z| = 1$. **2 Marks**

ii) If $f(z)$ is analytic and bounded in the entire z - plane then prove that $f(z)$ is constant.

b) Evaluate $\oint_c \frac{z^2-4}{z(z^2+9)}$ where $C$ is the circle $|z| = 1$ **5 Marks**

c) Prove that $\omega = \frac{i(z-i)}{z+i}$ maps the upper half of the z plane into the interior of the unit circle in $w$ - plane. 6 Marks

**4.** a) i) Evaluate $\left(\frac{\Delta^2}{E}\right) e^x \frac{Ee^x}{\Delta^2 e^x}$ 2 Marks

ii) Evaluate $\Delta^2 \cos 2x$ 2 Marks

b) Find a polynomial of $3^{rd}$ degree which takes the values 5 Marks

| | | | | | | |
|---|---|---|---|---|---|---|
| x | : | 3 | 4 | 5 | 6 | 7 |
| f(x) | : | 6 | 24 | 60 | 120 | 210 |

c) Given

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| x | : | 1 | 2 | 3 | 4 | 5 | 6 |
| f(x) | : | 1 | 8 | 27 | 64 | 125 | 216 |

Estimate $f(2.5)$ 6 Marks

**PART - B**

**5.** a) i) Find $L\{sin^2\ 4(t)\}$ 2 Marks

ii) If $L\{f(t)\} = F(s)$ then prove that
$L\{f'(t)\} = s\ F(s) - f(0)$ 2 Marks

b) Find $L\{f(t)\}$ where

$$f(t) = \begin{cases} 2, & 0 < t < 1 \\ t, & t > 1 \end{cases}$$ 5 Marks

c) Express $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 6, & t > 2 \end{cases}$

in terms of unit step function and find $L\{f(t)\}$. 6 Marks

**6.** a) i) Find $L^{-1}\left\{\frac{2s-1}{s^2-2s+10}\right\}$

ii) Find $L\ \{t \cdot cos\ at\}$ 2 Marks

b) If $L\{f(t)\} = F(s)$. Then prove that

$L\{\int_0^t f(u)du\} = \frac{F(s)}{s},\ \ S > 0$ 5 Marks

c) Solve $9y'' - 6y' + y = 0$ given that $y(0) = 3$ and $y'(0) = 1$ by Laplace transform method. 6 Marks

7. a) i) Use trapezoidal rule to evaluate $\int_0^1 e^x \, dx$ given

| $x$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|---|---|---|---|---|---|---|
| $y_x$ | 1 | 1.2214 | 1.4918 | 1.8221 | 2.2255 | 2.7183 |

**2 Marks**

ii) Show that the root of $x^3 - 3x - 5 = 0$ lies between 2 and 3. **2 Marks**

b) Using Simpson's $\frac{1}{3}$ rule, estimate the value of $\int_1^2 \frac{dx}{1+x^2}$ dividing the interval $(1, 2)$ into 4 equal parts. **5 Marks**

c) Find the root of the equation $x^3 - 9x + 1 = 0$ near $x = 3$ correct to three decimal places by Newton - Raphson method. **6 Marks**

8. a) i) Use Weddle rule to evaluate $\int_0^{0.6} \frac{dx}{1+x^2}$ given

| $x$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
|---|---|---|---|---|---|---|---|
| $y_x$ | 1 | 0.9901 | 0.9615 | 0.9174 | 0.8621 | 0.8000 | 0.7353 |

**2 Marks**

ii) By Euler's method find $y_1$ of $\frac{dy}{dx} = x^2 + y$ where $y = 0.94$ when $x = 0$, for $x = 0.1$ **2 Marks**

b) Using Picards method of successive approximation find the solution of $\frac{dy}{dx} = 1 + xy$, subject to the condition $y = 0$ when $x = 0$ upon third approximation. **5 Marks**

c) Solve $\frac{dy}{dx} = x + y^2$ with initial condition $y = 1$ when $x = 0$ for $x = 0.2$ using Runge - Kutta method. **6 Marks**

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