Third Year B.Sc. Degree Examination
Aug/Sept 2009
Directorate of Distance Education Course

MATHEMATICS : (PAPER - V)

Time : 3 Hours Max. Marks : 90

Note : Answer any SIX of the following.

PART - A

1. a) i) Separate into real and imaginary parts of exp $(5 + i\frac{\pi}{2})$ **2**

   ii) Find the derivative of $f(z) = z^2$ at 1+i. **2**

   b) Find the equation of the circle passing through the points 1+i, 2i, 1–i. Also find its centre and radius. **5**

   c) Find an analytic function whose imaginary part is $V = e^x (x \sin y + y \cos y)$. **6**

2. a) i) Show that $u = \cos x \cosh y$ is harmonic. **2**

   ii) Evaluate $\int_c z^2 dz$ where 'c' denotes the straight line path y=x from (0,0) to (1,1). **2**

   b) Show that $f(z) = e^z$ is analytic and $f^1(z)$ in terms of z. **5**

   c) Evaluate $\int_{(0,1)}^{(2,5)} (3x+y)dx + (2y-x)\,dy$ along

   i) the curve $y=x^2+1$

   ii) the line joining (0,1) and (2,5) **6**

3. a) i) Evaluate $\int_c \frac{\sin(\pi z^2) + \cos(\pi z^2)}{(z-1)(z-2)} dz$ **2**

   Where C : $|z| = 3$

   ii) Find the fixed points of the transformation $W = \frac{2z-1}{z}$ **2**

   b) State and prove Cauchy's integral formula for the first derivative of an analytic function. **5**

   c) Find a bilinear transformation which maps ∞, i, 0, onto 0, i, ∞ **6**

4. a) i) Evaluate $\Delta^3 e^{ax}$, where 'h' is the interval of differencing. **2**

   ii) Construct the forward difference table of the polynomial $f(x)=x^2+x+1$ for the values x=0(1)4. **2**

   b) Find a cubic polynomial which takes the values

x :	1	2	3	4	5
f(x) :	0	10	38	96	196

Using Newton's Gregory forward difference interpolation formula. **5**

c) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ of the function f(x) at x=1.5, given

x :	1.5	2	2.5	3	3.5	4
f(x) :	3.375	7	13.625	24	38.875	59

6

PART - B

5. a) i) Find $L\{e^t \cos t\}$. 2

ii) If $L\{f(t)\} = f(s)$ then prove that

$L\{e^{at} f(t)\} = f(s-a)$ 2

b) Find the laplace transform of the periodic function F(t) with period 2π such that

$$F(t) = \begin{cases} \sin t & , 0 < t < \pi \\ O & , \pi < t < 2\pi \end{cases}$$ 05

c) Express the function F(t) in terms of unit step function and find L{F(t)}, where

$$F(t) = \begin{cases} t^2 & ,0 < t < 2 \\ t-1 & ,2 < t < 3 \\ 7 & ,t > 3 \end{cases}$$ 6

6. a) i) Find $L^{-1}\left\{\frac{s-1}{(s-1)^2 + 2^2}\right\}$ 2

ii) Find L{F(t)} from the integral equation

$$F(t) = 4t - 3\int_0^t F(\beta) \sin(t-\beta)\, d\beta$$ 2

b) Show that $L^{-1}\left\{\frac{2a^3}{(s^2+a^2)^2}\right\} = \sin at - at \cos at$ 5

c) Solve the initial value problem

$x''(t) + 2x'(t) + x(t) = 3t\, e^{-t}$, given that $x(0)=4$ and $x'(0) = 2$. 6

7. a) i) Use Trapezoidal rule to evaluate $\int_0^6 y_x dx$, given that

x	0	1	2	3	4	5	6
y_x	0.135	0.158	0.169	0.179	0.192	0.214	0.230

2

ii) Show that the real root of the equation $x^3-x-4=0$ lies between 1 and 2. 2

b) Using Newton Raphson method to find a real root of the equation $x^3-3x-5=0$, carry out four iterations. 5

c) Using Simpson's $\frac{3th}{8}$ rule, evaluate $\int_{2}^{8} \frac{dx}{1+x}$ accurate upto three decimal places, dividing the interval (2,8) into six equal parts. 6

8. a) i) Evaluate : $\int_{0}^{0.6} \frac{dx}{1+x^2}$ from the following data using Weddle's rule.

x	0	0.1	0.2	0.3	0.4	0.5	0.6
y	1	0.9901	0.9615	0.9174	0.8621	0.8	0.7353

2

ii) Using Euler's method, solve $\frac{dy}{dx}$ = x+y, find y(0.2) given y=1 when x=0 and also h=0.2. 2

b) Solve : $\frac{dy}{dx} = x^2+y^2$, using Picard's method of successive approximations, given y(0)=0, upto third approximations, find $y^{(3)}$ at x=0.4. 5

c) Solve by using Runge-Kutta fourth order method, evaluate $\frac{dy}{dx} = \frac{x+y}{2}$ with the initial condition y(1)=3 with h=0.2 at x=1.2. 6

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