Third Year B.Sc. Degree Examination
Aug/Sept 2009
Directorate of Distance Education Course

MATHEMATICS (PAPER - III)

Time : 3 Hours — Max. Marks : 90

Note : Answer any SIX full questions of the following.

PART - A

1. a) i) If H is a normal subgroup of finite order then prove that
$$O\left(\frac{G}{H}\right) = \frac{O(G)}{O(H)}$$ 2

ii) Find whether the mapping $f : (z, +) \to (2z, +)$ defined by $f(x)=2x, \forall\, x \in z$ is a homomorphism or not. 2

b) Prove that a subgroup H of a group G is a normal subgroup of G iff $gHg^{-1}=H$, $\forall\, g \in G$. 5

c) If f is a homomorphism of a group G into a group G' with Kernel K, then prove that f(G) is isomorphic to factor group G/K. 6

2. a) i) In a ring R, prove that $a(b-c) = ab - ac \; \gamma \; a, b, c \in R$. 2

ii) Show that the set $S = \left\{ \begin{pmatrix} a & o \\ b & c \end{pmatrix} \,\middle|\, a, b, c \in \varphi \right\}$ is a subring of the ring $M_2(Z)$ of all 2x2 matrices. 2

b) Prove that a ring is without zero divisors iff the cancellation laws hold in it. 5

c) If $f : R \to R'$ is an isomorphism of the ring R onto R′ then prove that
i) isomorphic image of a commutative ring is commutative ii) isomorphic image of a ring with unity is a ring with unity. 6

3. a) i) Prove that $1-\sqrt{5}$ is not a unit in $Z(\sqrt{5})$ 2

ii) Find the polynomials q(x) & r(x) if $f(x) = x^5-x^3+3x-5$ & $g(x)=x^2+7$ 2

b) If a polynomial $f(x) \in F[x]$, where F is a field, is divided by x–a, then prove that the remainder is f(a). 5

c) Test the equation $5x^3+8x^2+6x-4 = 0$ for rational root. 6

4. a) i) Prove that the intersection of two normal subgroups of a group G is a normal subgroup of G. 2

ii) Prove that the Kernel homomorphis $f : R \to R'$ of rings is a subring of R. 2

b) Prove that a subgroup H of a group G is a normal a subgroup of G iff every right coset of H in G is a left coset of H in G. 5

c) If u & v are ideals of a ring R. Let uv be the set of all elements that can be written as finite sums of elements of the form uv where $u \in u$ & $v \in v$ show that uv is an ideal of R. 6

PART - B

5\. a) i) In any vectorspace prove that $c(-\alpha) = -c(\alpha)$ **2**

ii) Find the subspace spanned by the set $S = \{(3,0,0), (0,0,-2)\}$ in $v_3(R)$. **2**

b) If S & T are two subspaces of a vectorspace V(F) then prove that their linear sum

$S + T = \{\alpha+\beta \mid \alpha \in s, \beta \in T\}$ is also a subspace of v. **5**

c) Let $S = \{(1,2,1), (1,1,-1), (4,5,-2)$ is a subset of $V_3(R)$. Show that the vector $(2,-1,-8)$ is in L[s]. **6**

6\. a) i) Prove by an example that the union of two subspace need not be a subspace. **2**

ii) Do the vectors (1,2,1) & (2,1,1) form a basis for $V_3(R)$? Why? **2**

b) Prove that any two basis of a finite dimensional vectorspace V have the same finite number of elements. **5**

c) Find the basis and dimension of the subspace spanned by $S=\{(1,2,3), (3,1,0), (-2,1,3)\}$ in $V_3(R)$. **6**

7\. a) i) Prove that $T : V_2(R) \rightarrow V_3(R)$ defined by $T(x,y) = (x,o,y)$ is a linear transformation. **2**

ii) Prove that the range space of a linear transformation $T: V \rightarrow W$ is a subspace of W. **2**

b) Find the matrix of the linear transformation $T : V_2(R) \rightarrow V_3(R)$ defined by $T(-1,1) = (-1,0,2)$ & $T(2,1) = (1,2,1)$. **5**

c) State and prove rank-nullity theorem. **6**

8\. a) i) Find u_x and u_y for the function $u = \log \frac{x^2+y^2}{xy}$ **2**

ii) If $u = \frac{x^2+y^2}{\sqrt{x+y}}$ show that $xu_x+yu_y= \frac{3u}{2}$ **2**

b) If u = f(x,y) is a homogeneous function of degree n in x&y then prove that

$x^2 u_{xx} + 2_{xy} u_{xy} + y^2 U_{yy} = n(n-1)u$ **5**

c) Show that a rectangular solid of maximum volumes which can be inscribed in a sphere is a cube. **6**

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