Third Year B.Sc. Degree Examinations

December 2017

(Directorate of Distance Education)

MATHEMATICS

Paper – V: DSC 232: Mathematics

Time: 3 hrs] *[Max. Marks: 90*

Instructions to candidates:

1. *Answer any **SIX** of following:*
2. *Scientific calculator is allowed.*

PART – A

1. a) i) Find the real and imaginary parts of $e^{2+i\pi}$.

ii) Evaluate $\lim_{z\to 1+i}\left(z^2+2z\right)$. (2 + 2)

b) Show that $\arg\left[\dfrac{z-3}{z+3}\right]=2$ represents a circle and find its centre and radius. (5)

c) Find equation of the straight line joining the points $2+i,\ 3-2i$. (6)

2. a) i) Show that $w=z+e^z$ is analytic.

ii) Verify whether $f(z)=e^x\left[\cos y+i\sin y\right]$ is harmonic (2 + 2)

b) Show that $u=\sin x.\cosh y+2\cos x\sinh y+x^2-y^2+4xy$ is harmonic. Also determine corresponding analytic function. (5)

c) Prove that the real and imaginary parts of analytic function are harmonic. (6)

3. a) i) Evaluate $\int_{(0,3)}^{(2,4)}\left(2y+x^2\right)dx+\left(3x-y\right).dy$ along the parabola $x=2t,\ y=t^2+3$

ii) Find fixed points of the bilinear transformation $w=\dfrac{3z-4}{z}$ (2 + 2)

b) State and prove Cauchy's integral theorem (5)

c) Discuss the transformation $w=e^z$ (6)

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4. a) i) Prove that $\nabla = 1 - E^{-1}$

ii) If $\Delta^3 (1+\alpha x)(1-2x)(1+4x) = -144$ for $h = 1$ find α. (2 + 2)

b) From the following table estimate the number of persons in the income group Rs 20 – 25 (5)

Income per day	Under 10	10 – 20	20 – 30	30 – 40	40 – 50
No of Persons	20	45	115	210	115

c) For the data

x	– 2	– 1	0	1	2	3
y	0	0	6	24	60	120

compute $\left[\dfrac{dy}{dx}\right]_{x=2}$ and $\left[\dfrac{d^2y}{dx^2}\right]_{x=4.5}$ (6)

PART – B

5. a) i) Find $L[\cos at]$

ii) Find $L[\sin 7t.\ \sin 2t]$ (2 + 2)

b) Find Laplace transformation for the periodic function

$$f(t) = \begin{bmatrix} t & for & 0 < t < \pi \\ \pi - t & for & \pi < t < 2\pi \end{bmatrix} \text{ and } f(t + 2\pi) = f(t) \quad (5)$$

c) Express the function $f(t)$ in terms of unit step function and find its Laplace transformation.

$$f(t) = \begin{bmatrix} 2 & if & 0 < t < 1 \\ 1 & if & t \geq 1 \end{bmatrix} \quad (6)$$

6. a) i) Find $L\left[a^t\right]$

ii) Find $L^{-1}\left[\dfrac{s}{(s+4)^2}\right]$ (2 + 2)

b) Evaluate $L^{-1}\left[\dfrac{s}{(s^2+a^2)^2}\right]$ using convolution theorem (5)

c) Solve $y' - 5y = e^{5x}$ given $y(0) = 2$. (6)

7. a) i) Show that $\Delta - \nabla = \Delta \cdot \nabla$

ii) Evaluate $\Delta \tan^{-1} ax$ (2 + 2)

b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$, using Trapezoidal rule with $h = 1$ (5)

c) Evaluate $\int_0^1 \frac{dx}{1+x^2}$ dividing the range into 8 equal parts by using Simpsons ⅓rd rule and hence find the value of π. (6)

8. a) i) Evaluate $\int_{0.2}^{1.4} e^{2x}.dx$ using Weddle's rule with seven ordinates

ii) Find $f'(x)$ and $f''(x)$ for the function $x = 1.5$ given (2 + 2)

x	1.5	2	2.5	3	3.5	4
$f(x)$	3.37	7	13.62	24	38.87	59

b) Find solution of $\frac{dy}{dx} = 1 + xy$ using Picard's method of successive approximation subject to condition $y = 0$ when $x = 0$ up to 3rd approximation. Also obtain y when $x = 0.2$ (5)

c) Solve $\frac{dy}{dx} = xy$, given $y(1) = 2$ at $x = 1.2$ by using Runge – Kutta 4th order method. (6)

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