# Third Year B.Sc. Degree Examinations

## December 2017

*(Directorate of Distance Education)*

## MATHEMATICS

### Paper – III: DSC 230: Mathematics

*Time: 3 hrs]* *[Max. Marks: 90*

***Instructions to candidates:***

*Answer any **SIX** of the following.*

### PART – A

**1.** a) i) Define normal subgroup of a group

ii) If $H=\{1,-1\}$ is a subgroup of the multiplicative group $\{1,-1,i,-i\}$, then show that $H\Delta G$ where $G=\{1,-1,i,-i\}$. (2 + 2)

b) Prove that $H\Delta G$ iff $gHg^{-1}=H$, $\forall g\in G$, H is subgroup of G. (5)

c) State and prove fundamental theorem on Homomorphism of groups. (6)

**2.** a) i) Define a ring

ii) Define an integral domain and give an example. (2 + 2)

b) Prove that a ring is without zero divisors iff the cancellation laws hold in it. (5)

c) Prove that a commutative ring R with unity is a field iff R has no proper ideals. (6)

**3.** a) i) Define a polynomial ring and give an example.

ii) Find the product of the polynomials $\left(3x^{\circ}+4x+2x^2\right)$ and $1x^{\circ}+3x+4x^2+2x^3$ over the ring $\left(Z_5,+_5,\times_5\right)$. (2 + 2)

b) State and prove remainder theorem. (5)

c) Find the GCD of $x^3+2$ and $x^3+2x^2+2x+1$ over $Z_3$ and express it in the form $a(x)f(x)+b(x)g(x)$. (6)

**4.** a) i) If $f:\left(Z_1\ +\right)\rightarrow\left(Z_1\ +\right)$ is defined by $f(x)=x+1$ then verify whether $f$ is a homomorphism.

ii) Prove that homomorphic image of an abelian group is abelion. (2 + 2)

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b) If $f: G \to G'$ is a homomorphism and H is a cyclic subgroup of G then prove that $f(H)$ is also a cyclic subgroup of $G'$. (5)

c) If H is a normal subgroup of a group G. Then prove that $G/H$ is a homomorphic image of G with H as its Kernal. (6)

## PART – B

**5.** a) i) Define Vector Space.

ii) Prove that the subset $W = \{(x, y, z) / x - 3y + 4z = 0\}$ of the vector space $R^3$ is a subspace of $R^3$. (2 + 2)

b) Prove that the set of all ordered $n$ tuples of complex numbers forms a vector space over the field of complex numbers. (5)

c) Express the vector $(2, -1, -8)$ as a linear combination of the vectors $(1,2,1)$ $(1,1,-1)$, $(4,5,-2)$ . (6)

**6.** a) i) Find the subspace spanned by the vectors $(3,0,0)$ and $(0,0,-5)$ of the vector space $V_3(R)$.

ii) Show that the set $S = \{(1,1,1), (2,2,0), (3,0,0)\}$ is linearly dependent. (2 + 2)

b) Prove that any two bases of a finite dimensional vector space V have the same finite number of elements. (5)

c) Find a basis and dimension of the subspace spanned by the vectors $(2,-3,1), (3,0,1), (0,2,1), (1,1,1)$ of $V_3(R)$ (6)

**7.** a) i) Define Linear transformation

ii) If $T: U \to V$ is a linear transformation, then prove that $T(0) = 0'$, where 0 and $0'$ are zero vectors of U and V. (2 + 2)

b) Find the matrix of the linear transformation $T: V_2(R) \to V_3(R)$ such that $T(-1, 1) = (-1, 0, 2)$ and $T(2, 1) = (1, 2, 1)$. (5)

c) State and prove Rank – nullity theorem. (6)

**8.** a) i) If $u = \tan^{-1}\left(y/x\right)$ then find $u_x$, $u_y$.

ii) If $f = \sqrt{x^2 - y^2}$ the prove that $x.\frac{\partial t}{\partial x} + y.\frac{\partial t}{\partial y} = 1$ (2 + 2)

b) If $u = f(x+ay) + g(x-ay)$ then prove that $\frac{\partial^2 u}{\partial y^2} = a^2 \frac{\partial^2 u}{\partial x^2}$ (5)

c) Find the maxima and minima at the stationary points of the function $x^3 + y^3 - 3xy$. (6)

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