First Year B.Sc. Degree Examinations

December 2017

(Directorate of Distance Education)

MATHEMATICS

Paper – I: DSA 230: Mathematics

Time: 3 hrs] *[Max. Marks: 90*

Instructions to candidates:

*Answer any **SIX** full questions of the following choosing at least one from each part.*

PART – A

1. a) i) Find the remainder when $135 \times 74 \times 48$ is divided by 7.

ii) If there exists integers x and y such that $ax + by = 1$ then prove that $(a, b) = 1$ (2 + 2)

b) Solve the simultaneous congrunces $x \equiv 2 \pmod 3$ and $x \equiv 3 \pmod 5$. (5)

c) Prove that the relation "Congruence modulo m" is an equivalence relation in the set of integers. (6)

2. a) i) Define symmetric relation. Give an example.

ii) Let $A = \{-2, -1, 0, 1, 2\}$, $B = \{0, 1, 4\}$ Define $f : A \to B$ by $f(x) = x^2 \ \forall \ x \in A$, then prove that f is not one – one. (2 + 2)

b) If $f : X \to Y$ is a function from X into Y then for subsets $A, B \in X$ and $C, D \in Y$ then prove that .

i) $f(A \cup B) = f(A) \cup f(B)$

ii) $f(A \cap B) = f(A) \cap f(B)$ (5)

c) If $f : A \to B$ is a bijective map then prove that $f^{-1} : B \to A$ is also a bijective map and is unique. (6)

PART – B

3. a) i) Discuss the continuity of $f(x) = 3x^2 + 4x - 5$ at $x = 1$.

ii) Find the n^{th} derivative of a^{mx}. (2 + 2)

b) Examine the differentiability of the function

$$f(x) = \begin{cases} x^2 - 1 & if \quad x \geq 1 \\ 1 - x & if \quad x < 1 \end{cases} \quad \text{at} \quad x = 1$$ (5)

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c) If $y=\sin\left(m\sin^{-1}x\right)$ then show that $\left(1-x^2\right)y_{n+2}-\left(2n+1\right)xy_{n+1}+\left(m^2-n^2\right)y_n=0$. (6)

4. a) i) Find $\dfrac{ds}{dx}$ for the curve $y=a\cosh\left(x/a\right)$

ii) Find the pedal equation of the curve $r^n=a^n\sin n\theta$ (2 + 2)

b) Show that the pair of curves $r=a\left(1+\cos\theta\right)$ and $r=a\left(1-\cos\theta\right)$ intersect orthogonally. (5)

c) Show that the evaluate of the cycloid $x=a\left(\theta-\sin\theta\right),\ y=a\left(1-\cos\theta\right)$ is another cycloid . (6)

PART – C

5. a) i) Find the equation of the plane passing through $\left(4,\ 0,\ 6\right)$ and parallel to the plane $x+y+z=0$.

ii) Find the equation of the plane passing through the points $\left(6,\ 1,\ 5\right),\left(-6,\ 2,\ 7\right)$ and $\left(1,\ 2,\ 3\right)$. (2 + 2)

b) Find the equation of the plane passing through the point $\left(0,\ -4,\ 15\right)$ and the line $x=1-9t,\ \ y=2-3t,\ z=-2+51t$. (5)

c) Find the mutual position of the lines l_1 and l_2 given by .

$l_1: x=1+t,\ \ y=2+3t,\ \ z=8+7t$

$l_2: x=3+4s,\ \ y=5-2s,\ \ z=13-14s$ (6)

6. a) i) Find the center and radius of the sphere $x^2+y^2+z^2+2x-4y-6z+5=0$.

ii) Find the asymptotes of the curve $x^2+3xy+2y^2+3x-2y+1=0$ (2 + 2)

b) Find the position and nature of the double points of the curve $x^3+x^2+y^2-x-4y+3=0$. (5)

c) Find the volume generated by the revolution of ellipse $x=a\cos\theta$ and $y=b\sin\theta$ between $\theta=0$ and $\theta=\pi/2$. (6)

PART – D

7. a) i) Show that the following matrices are equivalent

$$A=\begin{bmatrix}1 & 2 & 3 & -4\\ 2 & -3 & 1 & 2\\ 5 & 0 & -2 & 7\end{bmatrix}\ \text{and}\ B=\begin{bmatrix}1 & 2 & -6 & -4\\ 9 & 8 & -20 & -9\\ 2 & -3 & -2 & 2\end{bmatrix}$$

ii) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$ (2 + 2)

b) Find the inverse of the matrix by using elementary row operations. (5)

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

c) Solve completely the system of equations.

$2x - y + 3z = 0,\ \ 3x + 2y + z = 0,\ \ x - 4y + 5z = 0$ (6)

8. a) i) Evaluate $\int \frac{dx}{5 - 3\cos x}$

ii) Evaluate $\int_{-\pi/2}^{\pi/2} \cos^6 x\, dx$ (2 + 2)

b) Evaluate $\int \frac{x\, dx}{(1 + x^2)\sqrt{1 - x^2}}$ (5)

c) Evaluate $\int_0^{\pi} x \sin^4 x \cos^6 x\, dx$ (6)

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