Q.P. Code – 50825

Third Year B.Sc., Degree Examinations, OCTOBER/NOVEMBER 2016

(Directorate of Distance Education)

(DSC 232) Paper V – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

1) *Answer any **SIX** of the following.*
2) *Scientific calculator is allowed.*

PART – A

1. (a) (i) Express $1 + i\sqrt{3}$ in the modulus argument form.

(ii) Find the equation of a circle given that centre $3 - 7i$ and radius = 8. **2 + 2**

(b) Find the equation of straight line through the points $z_1 = 1 - 2i$, $z_2 = 2 + i$. **5**

(c) Find the equation of the circle passing through the points $1 - 5i$, $2 + 10i$, $4 - i$. **6**

2. (a) (i) Find whether the function e^{x+iy} is analytic.

(ii) Prove that the function $\frac{1}{2}\log\left(x^2 + y^2\right)$ is harmonic. **2 + 2**

(b) Find the analytic function whose imaginary part is $\frac{x - y}{x^2 + y^2}$. **5**

(c) If $f(z) = u + iv$ is analytic and $u - v = (x - y)\left(x^2 + 4x + y^2\right)$, find $f(z)$ in terms of z. **6**

3. (a) (i) Evaluate $\oint_C \frac{dz}{z - 2}$ around the circle $|z - 2| = 4$.

(ii) Evaluate $\int_0^{1+i} z^2 dz$. **2 + 2**

(b) If $f(z)$ is analytic within and on a closed contour C and $z = a$ is an interior point of C then prove that $f'(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z - a)^2} dz$. **5**

(c) State and prove fundamental theorem of algebra. **6**

4. (a) (i) Show that $\left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{Ee^x}{\Delta^2 e^x} = e^x$.

(ii) Evaluate : $(E+2)(E-1)\left(e^x + x\right)$. **2 + 2**

(b) The following are the numbers of deaths in four successive ten years age groups. Find the numbers of deaths at 45–50 and 50–55. **5**

Age group :	25–35	35–45	45–55	55–65
Deaths :	13229	18139	24225	31496

(c) The following table gives the values of $\sin\theta$ for different values of θ :

θ	0°	10°	20°	30°	40°
$\sin\theta$	0.000	0.1736	0.3420	0.5000	0.6428

Find the value of cos 10°. **6**

PART – B

5. (a) (i) Find $L[\cos 3t \sin 2t]$.

(ii) Find $L\left[\sin^2 t\right]$. **2 + 2**

(b) Find the Laplace transform of $f(t) = \frac{Kt}{P}$ for $0 < t < p$ and $f(t+p) = f(t)$. **5**

(c) Evaluate $L^{-1}\left[\frac{(s+1)e^{-\pi s}}{s^2+s+1}\right]$ as a function of *t*. **6**

6. (a) (i) Find $L\left[e^{at}\left(2t^2 - 3t + 4\right)\right]$.

(ii) Find $L^{-1}\left[\frac{s+2}{s^2 - 4s + 13}\right]$. **2 + 2**

(b) Solve the integral equation $f(t) = at + \int_0^t f(u)\sin(t-u)\,du$. **5**

(c) Solve $y'' + y' - 2y = 3\cos 3t - 11\sin 3t$ given $y(0) = 0$ and $y'(0) = 6$. **6**

7. (a) (i) Prove that $(1+\Delta)(1-\nabla) \equiv 1$.

(ii) Given $u_0 = 1$, $u_1 = 11$, $u_2 = 21$, $u_3 = 28$ and $u_4 = 29$ find $\Delta^4 u_0$. **2 + 2**

(b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal Rule. **5**

(c) Using Simpson's $\left(\frac{1}{3}\right)^{\text{rd}}$ rule calculate $\int_4^{5.2} \log_e x\, dx$ by dividing the interval into 6 equal parts. **6**

8. (a) (i) Evaluate $\int_{0.2}^{1.4} y_x\, dx$ from the table using Weddle's rule.

x	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y_x	0.199	0.389	0.565	0.717	0.841	0.932	0.985

(ii) Using Picard's method of successive approximation find first approximation of $\frac{dy}{dx} = 1 + xy$ given that $y(0) = 0$. **2 + 2**

(b) Using Euler's modified method find an approximate value of y for $x = 0(0.2)0.6$ for $\frac{dy}{dx} = x + y$ given $y = 1$ when $x = 0$. **5**

(c) Solve $\frac{dy}{dx} = x + y^2$ with initial condition $y = 1$ when $x = 0$ for $x = 0.2(0.2)0.4$, using Runge-Kutta method. **6**
