Q.P. Code – 50823

Third Year B.Sc., Degree Examinations, OCTOBER/NOVEMBER 2016

(Directorate of Distance Education)

(DSC 230) Paper III – MATHEMATICS

Time : 3 Hours] *[Max. Marks : 90*

Instructions to Candidates :

Answer any ***SIX*** *of the following.*

PART – A

1. (a) (i) Define Factor Group with an example.

   (ii) Show that every quotient group of an abelian group is abelian. **2 + 2**

   (b) Show that the product of any two normal subgroups of a group is a normal subgroup of *G*. **5**

   (c) State and prove Cayley's theorem. **6**

2. (a) (i) Define integral domain with an example.

   (ii) Show that intersection of any two subrings is a sub ring. **2 + 2**

   (b) Prove that field has no proper ideals. **5**

   (c) Find all the principal and maximal ideals of the ring $(Z_6, +_6, X_6)$. **6**

3. (a) (i) Define units and prove that $9+4\sqrt{5}$ is a unit in $Z(\sqrt{5})$.

   (ii) Show that the polynomial x^2+x+1 is irreducible over Z_2. **2 + 2**

   (b) Find the GCD of $f(x)=4x^3-12x^2-15x-4$ and $g(x)=12x^2-24x-15$ over $Q[x]$ and express it as linear combination of $f(x)$ and $g(x)$. **5**

   (c) If $P(x)=a_0x^n+a_1x^{n-1}+\cdots+a_n$ be a polynomial with integral coefficients, then prove that any rational root of $P(x)=0$ must have the form $\frac{r}{s}$, where r/a_n and s/a_0 . **6**

4. (a) (i) Prove that every subgroup of an abelian group is a normal subgroup.

(ii) Is Z_5 field? Why? **2 + 2**

(b) If ϕ is a homomorphism of R into R' with Kernel $K(\phi)$, then prove that

(i) $K(\phi)$ is a sub group of R under addition.

(ii) If $a \in K(\phi)$ and $r \in R$ then ar and ra are in $K(\phi)$. **5**

(c) The homomorphism ϕ of a ring R into R' is an isomorphism iff $K(\phi) = (0)$. **6**

PART – B

5. (a) (i) In a vector space *V*, if $C \cdot \alpha = 0$ then show that either $C = 0$ or $\alpha = 0$.

(ii) Give an example to show that the union of two subspaces of a vector space *V* need not be a subspace of *V*. **2 + 2**

(b) In $V_3[Z_3]$, find how many vectors are spaned by (1, 2, 1) and (2, 1, 1)? Find all the vectors. **5**

(c) If '*n*' vectors span a vector space $V[F]$ and *r* vectors are linearly independent in *V*, then prove that $n \geq r$. **6**

6. (a) (i) Show that the set $S = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ is linearly independent in $V_3(R)$.

(ii) Do the vectors (1, 1, 0) and (0, 1, 1) form a basis for $V_3(Q)$? Why? **2 + 2**

(b) Show that all basis of any finite dimensional vector space *V* have the same finite number of vectors. **5**

(c) Show that the vectors (1, 0, –1), (1, 2, 1), (0, –3, 2) form a basis of $V_3(R)$. **6**

7. (a) (i) If $T : V_1(R) \to V_3(R)$ defined by $T(x) = (x, 2x^2, x^3)$, verify *T* is linear or not.

(ii) Find the linear transformation $T : R^3 \to R^3$ such that $T(1, 1) = (0, 1, 2)$ and $T(-1, 1) = (2, 1, 0)$. **2 + 2**

(b) If $T : R^3 \to R^3$ defined by $T(x, y, z) = (x + y, x - y, 2x + z)$. Find the range space, null space, rank and nullity of *T* and verify rank-nullity theorem. **5**

(c) Construct an orthonormal basis of $V_3(R)$ given that the basis vectors $\alpha_1 = (2, 1, 3)$, $\alpha_2 = (1, 2, 3)$ and $\alpha_3 = (1, 1, 1)$. **6**

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8. (a) (i) If $u = x\tan y + y\tan x$ then show that $\frac{\partial^2 u}{\partial x \cdot \partial y} = \frac{\partial^2 u}{\partial y \cdot \partial x}$.

(ii) If $u = \tan(y + ax) + (y - ax)^{3/2}$ then show that $\frac{\partial^2 u}{\partial x^2} - a^2 \frac{\partial^2 u}{\partial y^2} = 0$. **2 + 2**

(b) If $u = \tan^{-1}\left(\frac{x^2 + y^2}{x + y}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$. **5**

(c) Investigate the maxima and minima of the function $f(x, y) = 3x^2 - y^2 + x^3$. **6**
