KUVEMPU UNIVERSITY

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TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS: 2016-17

Course: M.Sc. MATHEMATICS (Previous)

Important Notes: (1) Students are advised to read the separate enclosed instructions before beginning the writing of assignments. (2) Out of 20 Internal Assignment marks per paper, 5 marks will be awarded for regularity (attendance) to Counseling/ Contact Programme classes pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper.

Answer all questions. Each question carries 05 marks.

PAPER I: ALGEBRA

1. a). Let G be a group in which i. $(ab)^3 = a^3b^3$ ii. $(ab)^5 = a^5b^5$ for all a, b in G. Show that G is abelian.

   b). Let p be a prime dividing o(G). Show that every sylow p-subgroup of G/K is of the form PK/K, where P is a sylow p-subgroup of G.

   c). Prove that the product of any two ideals of a ring R is also an ideal of R.

2. a). Show by an example that we can have a finite commutative ring in which every maximal ideal need not be prime.

   b). Let F be a field. If A = {(x, y, 0): x, y $\in$ F}, B = {(0, y, z): y, z $\in$F} be subspaces of $F^3(F)$, find the dimension of the subspace A+B.

   c). Obtain the eigen values, eigen vectors and eigen spaces of $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

3. a). Let T be a linear operator on a vector space V over F. If $W_1, W_2, \ldots, W_k$ are T-invariant subspaces of V, prove that $\sum_{i=1}^{k} W_i$ and $\bigcap_{i=1}^{k} W_i$ are T-invariant subspaces of V.

   b). If f(x) $\in$ F[x] is irreducible over F, then show that all its roots have the same multiplicity.

PAPER II: Analysis-I

1. a) If x, y are real numbers, $x > 0$, show that there exists a positive integer n, such that $x > y$

   b) For any $x > 0$, and for every positive integer n, show that there exist a unique $y \ni x = y^n$

   c) Prove that between any two real numbers there exists infinitely many rationals.

2. a) A metric space is called separable if it contains a countable dense subset. Show that R^k is separable.

   b) Let $f\colon [0,1) \to \mathbb{R}$ be a continuous function and suppose that $\lim_{x\to\infty} f(x)$ exists. Prove that f is uniformly continuous on $\mathbb{R}$.

   c) Give an example of an open cover of the segment $(0, 1)$ which has no finite subcover.

3. a) If f is a continuous mapping of a metric space X into metric space y, prove that $f(\bar{E}) \subset \overline{f(E)}$ for every set $E \subset X$. Show, by an example, that $f(\bar{E})$ can be a proper subset of $\overline{f(E)}$.

   b) A uniformly continuous function of a uniformly continuous function is uniformly continuous.

   c) Suppose α increases on [a, b], $a \le x_0 \le b$, α is continuous at x_0, $f(x_0) = 1$, and $f(x_0) = 0$ if $x \ne x_0$. Prove that $f \in R(\alpha)$ and that $\int f d\alpha = 0$.

PAPER III: ANALYSIS-II

1.a) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

b) Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of continuous functions which converges uniformly to a function f on a set E. Prove that $\lim_{n\to\infty} f_n(x_n) = f(x)$ for every sequence of points $x_n \in E$. such that $x_n \to x$ and $x \in E$. Is the converse of this true?

2.a) suppose $\{f_n\}, \{g_n\}$ are defined on E and (i) $\sum f_n$ has uniformly bounded partial sums

(ii) $g_n \to 0$ uniformly on E

$g_{1(x)} \geq g_{2(x)} \geq g_{3(x)} \geq \cdots$for every $x \in E$. Prove that $\sum f_n\, g_n$ converges uniformly on E.

b) Prove that the series $\sum_{n=1}^{\infty}(-1)^n \frac{x^2+n}{n^2}$ converges uniformly in every bounded interval, but does not converges absolutely for any value of x.

3.a) If the partial derivatives f_x and f_y exists and are bounded in a region $R \subset R^2$, then show that f is continuous in R.

b) Take $m = n = 1$ in the implicit function theorem and interpret the theorem graphically.

PAPER 1V: DIFFERENTIAL EQUATIONS

1.a) Explain the method of variation of parameters to find the solution of second order nonhomogeneous equations of the form $y'' + a_1 y' + a_2 y = b(x)$.

b) Find the solution of i) $y'' + y = \cos x$ ii) $y'' + 4y' - 5y = e^{2x}$

2.a) Find the solution of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}\sin x$, y(0) = 0, $y^1(0) = 1$ using Laplace transform method.

b) Find the integral surface of linear partial differential equation $x(y^2 + z) - y(x^2 + z)q = (x^2 - y^2)z$ which passes through the straight line $x + y = 0, z = 1$.

3.a) Reduce the partial differential equation $x^2\frac{\partial^2 z}{\partial x^2} + y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \sin(x)\log(y)$ in to its canonical form.

b) Obtain the solution of the partial differential equation $u_{xx} + u_{yy} = 0, 0 \leq x \leq a, 0 \leq y \leq b$ when subjected to the boundary conditions $u(0,y) = f_1(y),\ u(a,y) = f_2(y),\ u(x,0) = g_1(x),\ u(x,b) = g_2(x)$.

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