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TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS: 2015-16

Course: M.Sc. MATHEMATICS (Final Year)

Important Notes: (1) Students are advised to read the separate enclosed instructions before beginning the writing of assignments. (2) Out of 20 Internal Assignment marks per paper, 5 marks will be awarded for regularity (attendance) to Counseling/ Contact Programme classes pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper.

Answer all questions. Each question carries 05 marks.

PAPER V: COMPLEX ANALYSIS

1.a) Prove that every power series represents an analytic function within the radius of convergence.
   b) Verify that the function $f(Z) = Z^2$ satisfies the Cauchy-Riemann equations and determine the derivative $f'(z)$. .
   c) Find the inverse of a point a with respect to the circle $|z-c| = r$.

2. a) State and prove the Cauchy theorem for Triangle.
   b) Let $u(x,y) = x^2 - y^2 + 2x$, Find the conjugate function $v(x,y)$, such that $f(x,y) = u(x,y) + v(x,y)$, is an analytic function of z throughout the Z-Plane.
   c) Find all the roots of $\left(-8-8\sqrt{3i}\right)^{\frac{1}{4}}$.

3. a) State and prove the Cauchy theorem for Triangle.
   b) Define an index of a closed curve. Prove that index of a closed curve is an integer.
   c) Show that an isolated singularity “a” of f is removable iff $\lim_{z\to a}(z-a)f(z) = 0$.

PAPER VI: TOPOLOGY

1.a) Let A be a subset of a topological space X and let A′ be the set of all limit points of A. Prove that $\overline{A} = A \cup \overline{A}$.
   b) Prove that the space R^w, the countably infinite product of R in the box topology, is not metrizable.

2.a) Define a compact space. Show that a closed subspace of a compact space is compact and the image of a compact space under a continuous map is compact.
   b) Let X be a topological space, $A \subset X$. Show that
      I. If X is Hausdorff and A is compact, then A is closed.
      II. If X is compact and A is closed, then A is compact.

3.a) Prove that the space R_l , the real line with the lower limit topology is Lindelof and separable but not second countable.

b) i). Let $X = \{1,2,3,4\}$. Let $A = \{\{1,2\},\{2,4\},\{3\}\}$. Determine the topology on X generated by the elements of A and hence determine the base for this topology.

ii). Let $\tau_1 = \{\emptyset, \{1\}, X_1\}$ be a topology on $X_1 = \{1,2,3\}$ and

$\tau_2 = \{\emptyset, X_2, \{a\}, \{b\}, \{a,b\}, \{c,d\}, \{a,c,d\}, \{b,c,d\}\}$ be a topology for $X_2 = \{a,b,c,d\}$

Find a base for the product topology τ.

PAPER VII: MEASURE THEORY AND FUNCTIONAL ANALYSIS

1. a). Define outer measure. Justify whether the outer measure is countably additive or not.

b). Give an example of a sequence of sets $\{E_i\}$ with $E_i \supseteq E_{i+1}, m^*(E_i) < \infty$, and $m^*(\cap E_i) < \lim M^* E_i$.

c). Define the Lebesgue integral. Consider $(x) = \sum_{n=1}^{200} \frac{1}{n^6} \chi_{\left[0,\frac{n}{200}\right]}(x)$, $x \in [0,1]$, where χ is the characteristic function. Find the Lebesgue integral of f on [0,1].

2. a).Consider the function $f(x) = \begin{cases} [x]\cos(x) & if\ x\ rational \\ x^2 \sin(x) & if\ x\ is\ irrational \end{cases}$, where [x] denotes the greatest integer function. Justify whether f is a measurable function or not.

b). Show that C[a, b] is not complete under integral metric.

c). Define a separable space. Give an example of a non-separable subspace of a separable space.

3. a). Show that continuity and boundedness are equivalent on a normed linear space.

b). Show that all norms are equivalent on a finite dimensional normed linear space.

c). Let X be a Banach space. Prove that X is reflexive ⇔ X* is reflexive

PAPER VIII: NUMERICAL ANALYSIS

1.a) Describe Jacobi and Gauss-Seidel iteration method to solve the linear system of equations and hence discuss their convergence analysis.

b) Use (a) to extract the quadratic factor of the form $x^2 + px + q$ from the polynomial $x^4 + x^3 + x^2 + x + 1 = 0$ choose $p = 1, q = 1$.

2.a) Fit both linear and quadratic curve for the function $f(x) = 2 - x + x^2$ over an interval $[0,1]$ with respect to the weight function $W(x) = 1$ using the least square approximation method .

b) Find the cubic spline interpolation polynomial for the following data:

x	0	1	2	3
y	1	-2	1	16

with $S''(0) = S''(3) = 0$.

3.a) Solve an initial value problem $\frac{dy}{dx} = 2x(1+y)$, $y(0) = 0$ in the range $0 < x < 1$ using modified Adams Predictor Corrector method, choose h = 0.2.

b) Solve the equation $u_{tt} - u_{xx} = 0$, when subjected to the following conditions

$u(x,0) = \sin^3 \pi x, u_t(x,0) = 0, u(0,t) = 0, u(1,t) = 0$, *choose* $h = \frac{1}{4}$ $and\ k = 1/8$.

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