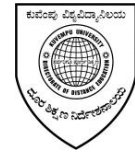




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TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS: 2015-16
Course: M.Sc. MATHEMATICS (Final Year)

Important Notes: (1) Students are advised to read the separate enclosed instructions before beginning the writing of assignments. (2) Out of 20 Internal Assignment marks per paper, 5 marks will be awarded for regularity (attendance) to Counseling/ Contact Programme classes pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper.

Answer all questions. Each question carries 05 marks.

PAPER V: COMPLEX ANALYSIS

1. a) Prove that every power series represents an analytic function within the radius of convergence.
b) Verify that the function $f(Z) = Z^2$ satisfies the Cauchy-Riemann equations and determine the derivative $f'(z)$.
c) Find the inverse of a point a with respect to the circle $|z - c| = r$.
2. a) State and prove the Cauchy theorem for Triangle.
b) Let $u(x, y) = x^2 - y^2 + 2x$, Find the conjugate function $v(x, y)$, such that $f(x, y) = u(x, y) + v(x, y)$, is an analytic function of z throughout the Z -Plane.
c) Find all the roots of $(-8 - 8\sqrt{3}i)^{\frac{1}{4}}$.
3. a) State and prove the Cauchy theorem for Triangle.
b) Define an index of a closed curve. Prove that index of a closed curve is an integer.
c) Show that an isolated singularity “ a ” of f is removable iff $\lim_{z \rightarrow a} (z - a)f(z) = 0$.

PAPER VI: TOPOLOGY

1. a) Let A be a subset of a topological space X and let A' be the set of all limit points of A . Prove that $\bar{A} = A \cup A'$.
b) Prove that the space R^w , the countably infinite product of R in the box topology, is not metrizable.
2. a) Define a compact space. Show that a closed subspace of a compact space is compact and the image of a compact space under a continuous map is compact.
b) Let X be a topological space, $A \subset X$. Show that
 - I. If X is Hausdorff and A is compact, then A is closed.
 - II. If X is compact and A is closed, then A is compact.
3. a) Prove that the space R_l , the real line with the lower limit topology is Lindelof and separable but not second countable.

- b) i). Let $X = \{1,2,3,4\}$. Let $A = \{\{1,2\}, \{2,4\}, \{3\}\}$. Determine the topology on X generated by the elements of A and hence determine the base for this topology.
- ii). Let $\tau_1 = \{\emptyset, \{1\}, X_1\}$ be a topology on $X_1 = \{1,2,3\}$ and $\tau_2 = \{\emptyset, X_2, \{a\}, \{b\}, \{a, b\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ be a topology for $X_2 = \{a, b, c, d\}$. Find a base for the product topology τ .

PAPER VII: MEASURE THEORY AND FUNCTIONAL ANALYSIS

- Define outer measure. Justify whether the outer measure is countably additive or not.
 - Give an example of a sequence of sets $\{E_i\}$ with $E_i \supseteq E_{i+1}$, $m^*(E_i) < \infty$, and $m^*(\cap E_i) < \lim M^*E_i$.
 - Define the Lebesgue integral. Consider $(x) = \sum_{n=1}^{200} \frac{1}{n^6} \chi_{[0, \frac{n}{200}]}(x)$, $x \in [0,1]$, where χ is the characteristic function. Find the Lebesgue integral of f on $[0,1]$.
- Consider the function $f(x) = \begin{cases} [x] \cos(x) & \text{if } x \text{ rational} \\ x^2 \sin(x) & \text{if } x \text{ is irrational} \end{cases}$, where $[x]$ denotes the greatest integer function. Justify whether f is a measurable function or not.
 - Show that $C[a, b]$ is not complete under integral metric.
 - Define a separable space. Give an example of a non-separable subspace of a separable space.
- Show that continuity and boundedness are equivalent on a normed linear space.
 - Show that all norms are equivalent on a finite dimensional normed linear space.
 - Let X be a Banach space. Prove that X is reflexive $\Leftrightarrow X^*$ is reflexive

PAPER VIII: NUMERICAL ANALYSIS

- Describe Jacobi and Gauss-Seidel iteration method to solve the linear system of equations and hence discuss their convergence analysis.
 - Use (a) to extract the quadratic factor of the form $x^2 + px + q$ from the polynomial $x^4 + x^3 + x^2 + x + 1 = 0$ choose $p = 1, q = 1$.
- Fit both linear and quadratic curve for the function $f(x) = 2 - x + x^2$ over an interval $[0, 1]$ with respect to the weight function $W(x) = 1$ using the least square approximation method.
 - Find the cubic spline interpolation polynomial for the following data:

x	0	1	2	3
y	1	-2	1	16

with $S''(0) = S''(3) = 0$.
- Solve an initial value problem $\frac{dy}{dx} = 2x(1 + y)$, $y(0) = 0$ in the range $0 < x < 1$ using modified Adams Predictor Corrector method, choose $h = 0.2$.
 - Solve the equation $u_{tt} - u_{xx} = 0$, when subjected to the following conditions $u(x, 0) = \sin^3 \pi x$, $u_t(x, 0) = 0$, $u(0, t) = 0$, $u(1, t) = 0$, choose $h = \frac{1}{4}$ and $k = 1/8$.
