KUVEMPU UNIVERSITY

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TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS: 2015-16

Course: M.Sc. MATHEMATICS (Previous)

Important Notes: (1) Students are advised to read the separate enclosed instructions before beginning the writing of assignments. (2) Out of 20 Internal Assignment marks per paper, 5 marks will be awarded for regularity (attendance) to Counseling/ Contact Programme classes pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper.

Answer all questions. Each question carries 05 marks.

PAPER -I: ALGEBRA

1. a) Define Normalizer and Centralizer of a group. If G is the group of all 2×2 non-singular matrices over R, find the centre of G.
   b) Find a Sylow 3-subgroup of S_9 .
   c) Show that Z[$\sqrt{2}$] is a Euclidean domain.

2. a) Show that the ideal $\langle x \rangle$ of Z$\langle x \rangle$ is a prime ideal but not maximal.
   b) Let V be the vector space of real valued functions $y = f(x)$ satisfying $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6y = 0$. Prove that V is a three-dimensional vector space over R. Define $\langle f, g \rangle = \int_{-\infty}^{0} fg\, dx$ in V. Find an orthogonal basis of V over R.
   c) Prove that a finite dimensional vector space has dimension $'n'$ iff $'n'$ is the maximum number of linearly independent vectors in any subset of V.

3. a) Obtain the eigen values, eigen vectors and eigen spaces of $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.
   b) Let T be a linear operator on R^2, the matrix of which in the standard ordered basis is $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$. Let W_1 be the subspaces of R^2 spanned by $(1,0)$. Prove that
   i) W_1 is invariant under T.
   ii) there exists no T-invariant subspace W_2 of R^2 such that $R^2 = W_1 \oplus W_2$.
   c) Find the degree of the minimal splitting field of $x^4 + 2$ over Q.

PAPER- II: Analysis-I

1. (a) Construct a bounded set of real numbers with exactly three limit points.
   (b) Prove that complement of E^0 is the closure of the complement of E, where E^0 is the set of all interior points of a set E.
   (c) Prove that every infinite set contains a countably infinite subset.
2. (a) Prove that if K is compact and C is closed in $\mathfrak{R}^k$ then $K + C$ is closed.
   (b) Let $f\colon [0,1) \to \mathbb{R}$ be a continuous function and suppose that $\lim_{x\to\infty} f(x)$ exists. Prove that f is uniformly continuous on $\mathbb{R}$.

3. (a) If $|f| \in \mathcal{R}(\alpha)$ on $[a, b]$, does $f \in \mathcal{R}(\alpha)$ on $[a, b]$? prove your claim.

(b) i) If $f_1, f_2 \in \mathcal{R}(\alpha)$ on $[a, b]$ then show that $f_1 o\, f_2 \in \mathcal{R}(\alpha)$ on $[a, b]$.

ii) Let $f(x) = \begin{cases} x^2 sin\frac{1}{x} & if \;\; 0 < x \leq 1, \\ 0 & if \;\; x = 0. \end{cases}$ Show that $f \in BV[0,1]$.

PAPER- III: ANALYSIS-II

1. a) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.
   b) Let $\{f_n\}_{n=1}^{\infty}$ be a sequence of continuous functions which converges uniformly to a function f on a set E. Prove that $\lim_{n\to\infty} f_n(x_n) = f(x)$ for every sequence of points $x_n \in E$. such that $x_n \to x$ and $x \in E$. Is the converse of this true?

2. a) suppose $\{f_n\}, \{g_n\}$ are defined on E and (i) $\sum f_n$ has uniformly bounded partial sums, (ii) $g_n \to 0$ uniformly on E, $g_{1(x)} \geq g_{2(x)} \geq g_{3(x)} \geq \cdots$ for every $x \in E$. Prove that $\sum f_n\, g_n$ converges uniformly on E.
   b) Prove that the series $\sum_{n=1}^{\infty}(-1)^n \frac{x^2+n}{n^2}$ converges uniformly in every bounded interval, but does not converges absolutely for any value of x.

3. a) If the partial derivatives f_x and f_y exists and are bounded in a region $R \subset R^2$, then show that f is continuous in R.
   b) Take $m = n = 1$ in the implicit function theorem and interpret the theorem graphically.

PAPER- IV: DIFFERENTIAL EQUATIONS

1. a) Determine whether the following functions are linearly dependent or independent

   i) $\phi_1 = \cos x$ and $\phi_2 = 3(e^{ix}+e^{-ix})$, ii) $\phi_1 = \sin x$ and $\phi_2 = e^{ix}$.

   b) Check for the oscillations or non-oscillations of the differential equation:

   $t^2x'' + tx + (t^2 - \alpha^2)x = 0.$

2. a) Find series solution of the following differential equation about $x = 0$

   $y'' + 3x^2y' - xy = 0.$

   b) Find the general solution of $(1 - xu)u_x + y(2x^2 + u)u_y = 2x(1 - xu)$, and then that solution which satisfies $u = e^y$ on $x = 0$.

3. a) Reduce the following equation into its respective canonical form;

   $y^2u_{xx} + 2xyu_{xy} + (x^2 + 4x^4)u_{yy} = \frac{2y^2}{x}u_x + \frac{1}{y}(y^2 + x^2 + 4x^4)u_y.$

   b) Find the solution of $u_t = \alpha^2 u_{xx} + \sin(3\pi x)$, $0 < x < 1, t \geq 0$ when subjected to the boundary conditions $u(0, t) = u(1, t) = 0$ with an initial condition $u(x, 0) = \sin(\pi x)$ using eigenfunction expansion method.

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