KUVEMPU UNIVERSITY

OFFICE OF THE DIRECTOR

DIRECTORATE OF DISTANCE EDUCATION

Jnana Sahyadri, Shankaraghatta – 577 451, Karnataka

Phone: 08282-256426; Fax: 08282-256370; Website: www.kuvempuuniversitydde.org

E-mails: ssgc@kuvempuuniversity.org; info@kuvempuuniversitydde.org

TOPICS FOR INTERNAL ASSESSMENT ASSIGNMENTS: 2015-16

Course: M.Sc. MATHEMATICS (Final Year)

Important Notes: (1) Students are advised to read the separate enclosed instructions before beginning the writing of assignments. (2) Out of 20 Internal Assignment marks per paper, 5 marks will be awarded for regularity (attendance) to Counseling/ Contact Programme classes pertaining to the paper. Therefore, the topics given below are only for 15 marks each paper.

Answer all questions. Each question carries 05 marks.

PAPER V: COMPLEX ANALYSIS

1. a) Find the inverse of a point 'a' with respect to the circle $|z-c|=r$.

   b) Prove that $f(z)$ analytic in a domain D such that $f(D)\subset D_1$. If a function g is analytic in the domain D_1 and $F(z)=(gf)(z)$, then show that $F(z)$ is analytic in D.

   c) Let $f(z)$ be an analytic function. Show that

   $$(i)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2=4\left|f'(z)\right|^2. \qquad (ii)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|Ref(z)|^2=2\left|f'(z)\right|^2.$$

2. a) Find the images of the infinite strip $0<y<1/2c$ under the map $W=\frac{1}{z}$.

   b) State and prove the Cauchy theorem for Triangle.

3. a) Prove or disprove $Res_{z=\infty}f(z)=-\lim_{z\to\infty} zf(z)$.

   b) Using Calculus of Residues, Evaluate the following $i)\int_0^{\pi}\frac{d\theta}{5-4\cos\theta}$, $ii)\int_0^{\infty}\frac{dx}{(1+x^2)^3}$.

PAPER -VI: TOPOLOGY

1. a) If T_1, T_2 and T_k denotes usual, Lower limit and $K-$Topologies on R_1, R_2 and R_k respectively, show that T_2 and T_k are strictly finer than T but are not comparable with one another.

   b) Let X be Topological space, $A,B\subset X$, show that

   (i) $\overline{A\cup B}=\bar{A}\cup\bar{B}$, (ii) $\overline{A\times B}=\bar{A}\times\bar{B}$,

   (iii) If $\bar{A}=\bar{B}$, is it true that $A=B$?, (iv) If $A\subset Y\subset X$, show that $\overline{A_Y}=\bar{A}\cap Y$.

2. a) Show that $R\times R$ in the dictionary order topology is metrizable.

   b) Using the closed set formulation of continuity, show that the following are closed subset of R^2, (i)$A=\{x\times y: xy=1\}$, (ii)$B=\{x\times y: x^2+y^2\le 1\}$.

3. a) If R has the topology consisting of all sets A such that $R-A$ is either countable or all of R, is $[0,1]$ a compact subspace?

   b) Show that a connected metric space having more than one point is uncountable.

PAPER- VII: MEASURE THEORY AND FUNCTIONAL ANALYSIS

1. a) Define a measurable set. Prove that the interval (a, ∞) is a measurable. Deduce that every Borel-Set is measurable.

   b) Find the Dini-Derivatives of $f(x) = \begin{cases} x \sin 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$ at $x = 0$.

   c) If f is absolutely continuous on $[a, b]$, then show that $f \in BV[a, b]$ and that f is an N-function.

2. a) Define a metric space. Show that $c[a, b]$ with the integral metric is not complete.

   b) Prove that the set of Rationals is of 1^{st} category and Irrationals is of 2^{nd} category.

   c) Explain compact and sequential compact metric spaces with an example.

3. a) Define a l_p space $(1 \leq p \leq \infty)$ and show that l_p is a complete normed linear space.

   b) State Halin Banach theorem. If M is a closed linear subspace of a normed linear X and let $x_0 \notin M$. If $d = d(x_0, M)$, then show that there exists a functional $f_0 \in X^*$ such that $f_0(M) = 0$, $f_0(x_0) = 1$ and $||f_0|| = 1/d$.

   c) If dual X^* of a normed linear space X is separable, Show that X is saparable, Is the converse true? Justify.

PAPER- VIII: NUMERICAL ANALYSIS

1. a) Perform two iterations of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $9x^4 + 30x^3 + 34x^2 + 30x + 25 = 0$, choose $p = q = 1$.

   b) Employ House-Holder method to reduce the following matrix into a real symmetric tri-diagonal matrix. Given $A = \begin{pmatrix} 3 & 1 & -5 \\ 1 & 3 & 3 \\ -5 & 3 & 3 \end{pmatrix}$.

2. a) Fit a quadratic curve to the following data using least square approximation method.

x	0	1	2	3	4
y	0	8.41	18.18	4.23	-30.27

   b) Solve the equation $x^2y'' + xy' + (x^2 - 4)y = 0$ when subjected to boundary conditions $y(0) = 0$, and $y(1) = 1$, using Finite difference method for $h = 0.5, 0.25, 0.2$ method.

3. a) Obtain the cubic spline interpolation polynomial for the function defined by the data

x	0	1	2	3
y	-10	-20	-10	140

   with $s''(0) = s''(3) = 1$.

   b) Apply Crank-Nicolson implicit formula to solve heat conduction equation

$$\begin{cases} u_t = 9e^{-t}u_{xx} - 4u, & 0 < x < \pi, t \geq 0, \\ u(x, 0) = 6\sin(x), & \forall\, 0 \leq x \leq \pi, \\ u(0, t) = 0 = u(\pi, t), & \forall\, t \geq 0. \end{cases} \quad \text{with } h = \frac{1}{4}, k = 0.01.$$

-----------0-----------